Sample-path large deviations for processes with heavy-tailed increments and applications

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Large deviations theory is a powerful framework in order to analyze and estimate rare events that arise in stochastic systems. Nevertheless, the classical large deviation results lack the power to deal with systems in which the underlying uncertainties are heavy-tailed. In this talk, I will present new sample path large deviation results for Lévy processes and random walks with heavy-tailed Weibull increments. Our results consist of an extended LDP in the Skorohod space w.r.t. the J_1 topology and an LDP w.r.t. the M_1' topology, and they hold for the two processes. Applications on queueing theory and large deviations for additive functionals of Markov random walks show the potential of our results. Specifically, we answer an open problem posed by Sergey Foss regarding the queue length asymptotics of many server queues with heavy-tailed Weibull service times. In that regard, I will provide insights on how a large queue length occurs and highlight differences with the well-studied regularly varying case. Lastly, I will present our sample-path LDP for unbounded additive functionals of the reflected random walk (r.r.w.) with light-tailed increments. Our technique hinges on a suitable decomposition of the Markov chain in terms of regeneration cycles. At each regeneration cycle, the accumulated area of the reflected random walk can be seen as a heavy-tailed increment. The LDP for unbounded additive functionals of the r.r.w. holds in the Skorokhod space equipped with the M_1' topology.